# Structured Encryption and Leakage Suppression

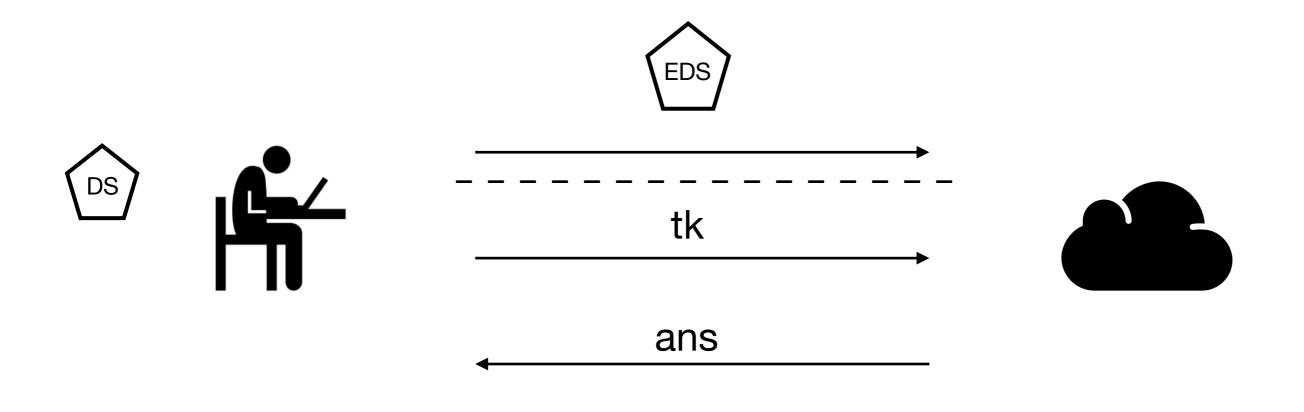
Tarik Moataz

Part I is a joint work with Seny Kamara and Olya Ohrimenko

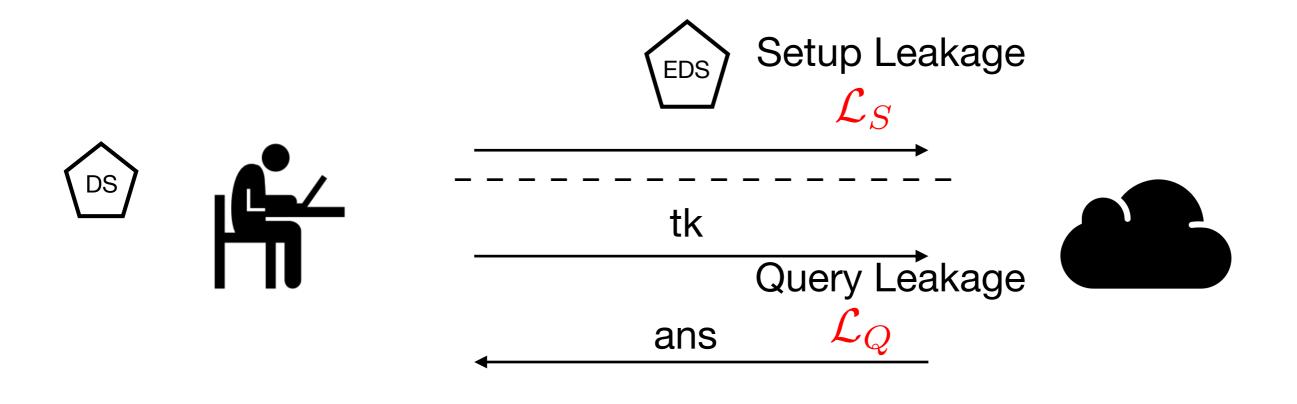
Part II is a joint work with Seny Kamara







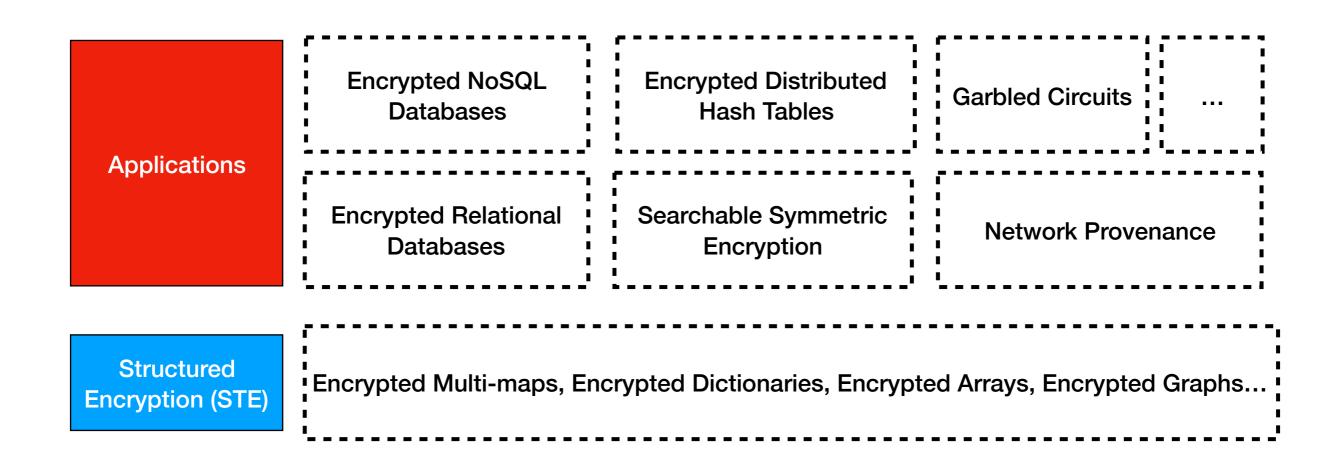
Setup 
$$\begin{bmatrix} 1^k, & \bigcirc \end{bmatrix}$$
 tk  $\leftarrow$  Token  $\begin{bmatrix} 0, & q \end{bmatrix}$ 

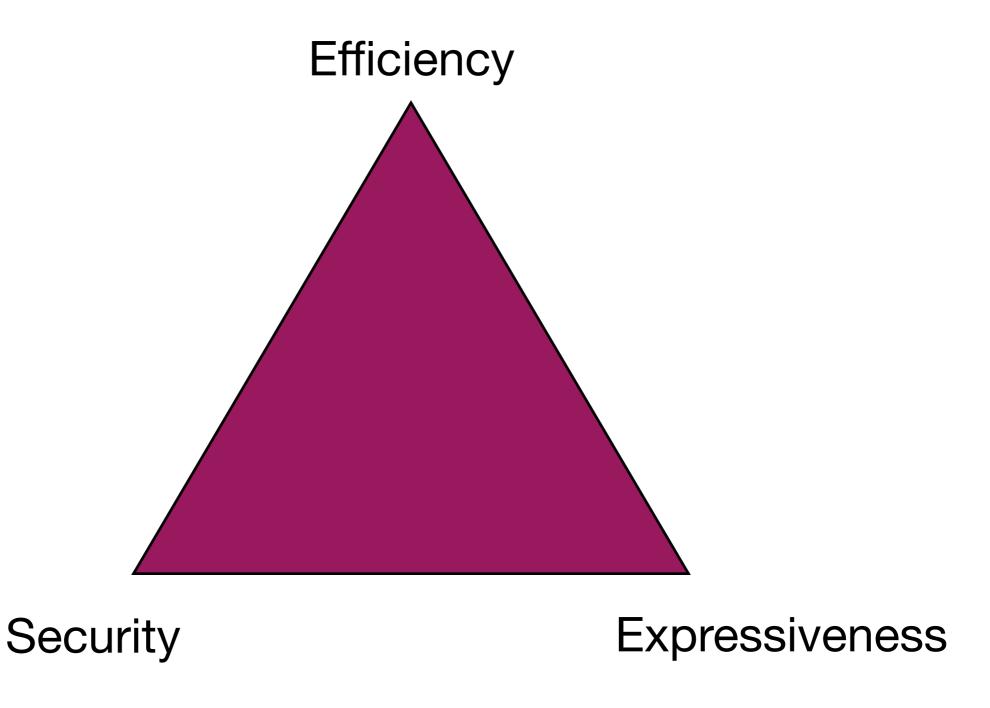


Setup 
$$\begin{bmatrix} 1^k, \\ \end{bmatrix}$$
  $\begin{bmatrix} 1^k, \\ \end{bmatrix}$   $\begin{bmatrix} 1^k, \\ \end{bmatrix}$ 

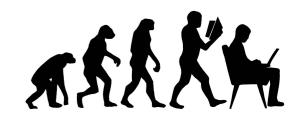
An STE scheme is  $(\mathcal{L}_S, \mathcal{L}_Q)$ -secure if

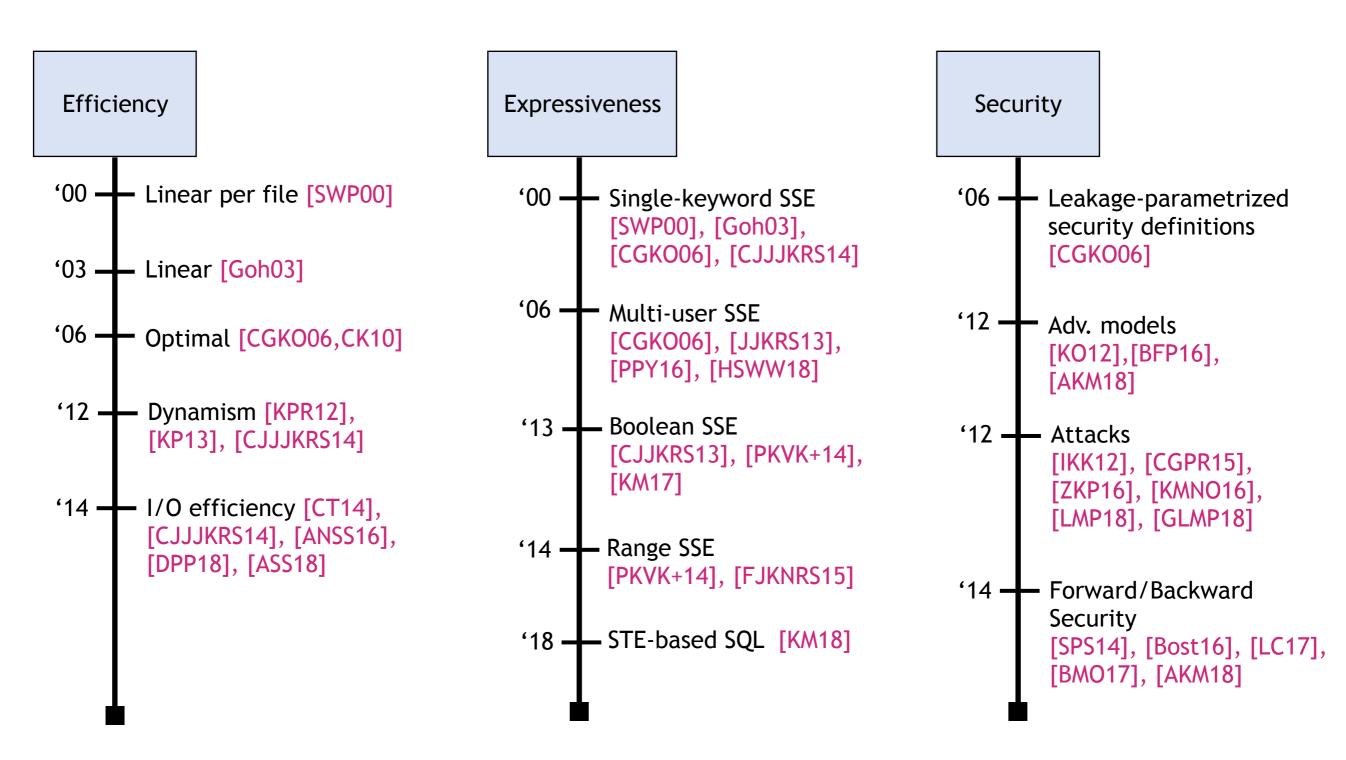
- It reveals no information about the structure beyond  $\mathcal{L}_S$
- It reveals no information about the structure and queries beyond  $\mathcal{L}_Q$



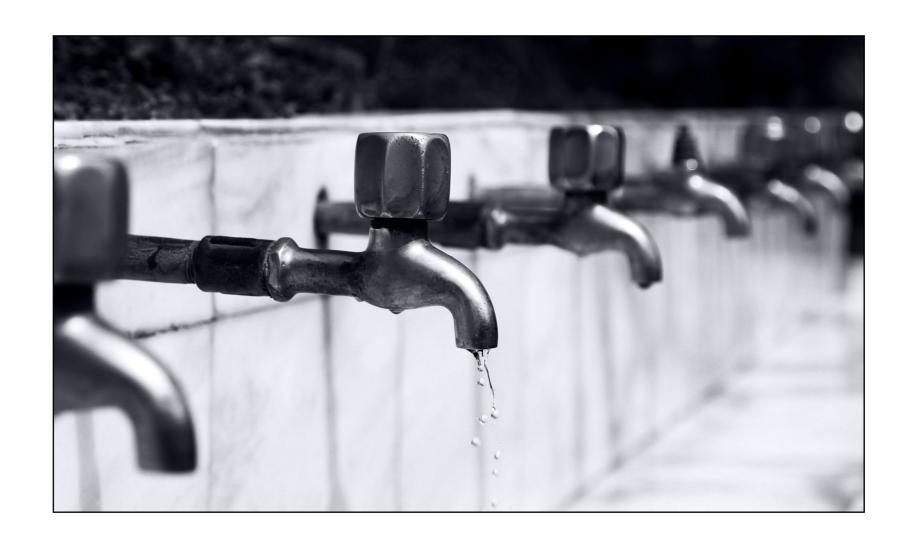


#### Structured Encryption Evolution





## What about Leakage?



#### What about Leakage?



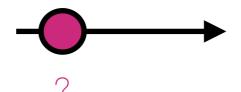




Cryptanalysis



Measure



Suppression





Cryptanalysis

Def: Given a leakage profile, design attacks to recover the queries or the data under some assumptions

Goal: empirically learn the impact of a leakage pattern in real-world

Limitations: the gap between assumptions and reality can get wide



Measure

Def: Given a leakage profile, quantify (e.g., in bits) a specific leakage pattern

Goal: theoretically compare between leakage patterns

Limitations: (maybe) no possible total order (work in progress!)



Suppression

Def: Given a leakage profile, design a compiler or a transform to suppress a specific leakage pattern

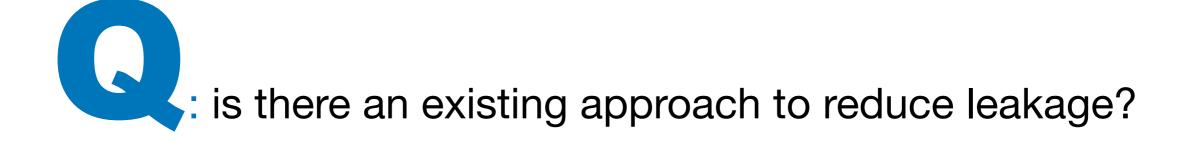
Goal: develop tools to suppress various leakage patterns

Limitations: introducing some overhead

# Part 1\*

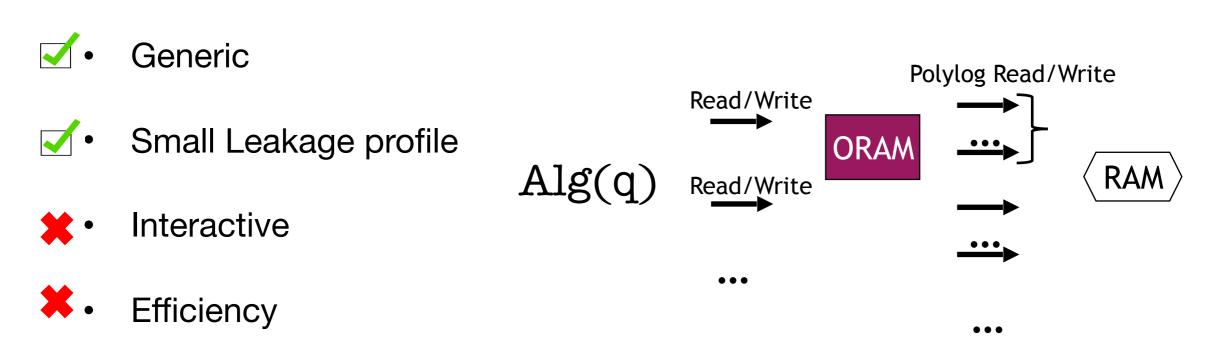
### Suppressing Leakage

\*joint work with Seny Kamara and Olya Ohrimenko https://eprint.iacr.org/2018/551

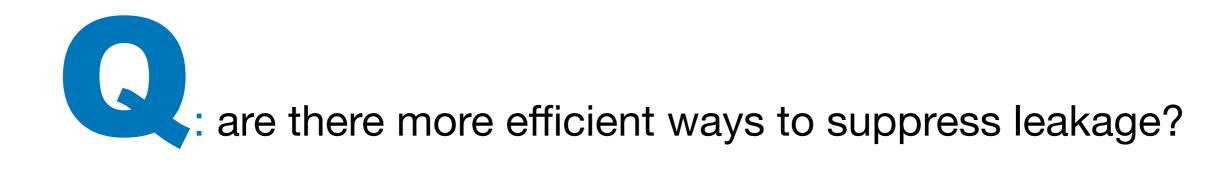


#### **Existing Approaches**

ORAM Simulation [GO96], [SvDSFRD13]



- Garbled RAM [LO13], [GHLORW14]
- Custom Schemes [WNLCSSH14], [BM16]



#### Background Modeling Leakage

- qeq : query equality
  - search pattern
- did: data identity
- req : response equality
- rid : response identity
  - access pattern

- qlen: query length
- rlen: response length
  - volume pattern
- mqlen: maximum query length
- mrlen: maximum response length
- srlen: sequence response length
- dsize: data size

#### Background

#### Non-Repeating Sub-Pattern

#### Non-repeating sub-pattern

$$\mathsf{patt}(\mathsf{DS},q_1,\cdots,q_t) = \left\{ \begin{array}{ll} \mathsf{nrp}(\mathsf{DS},q_1,\cdots,q_t) & \text{ if } q_i \neq q_j, \forall i,j \in [t] \\ \mathsf{rp}(\mathsf{DS},q_1,\cdots,q_t) & \text{ otherwise.} \end{array} \right.$$

#### Example

$$\operatorname{\mathsf{qeq}}(\mathsf{DS},q_1,\cdots,q_t) = \left\{ \begin{array}{ll} \bot & \text{if } q_i \neq q_j, \forall i,j \in [t] \\ \operatorname{\mathsf{rp}}(\mathsf{DS},q_1,\cdots,q_t) & \text{otherwise.} \end{array} \right.$$

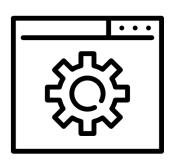
#### Leakage Suppression

#### Through Compilation



$$\begin{array}{c} STE \\ \Lambda = \left(\mathcal{L}_{S}, \mathcal{L}_{Q} = \left(\mathsf{patt}_{1}, \mathsf{p}\right)\right) \end{array} \longrightarrow \begin{array}{c} \mathsf{Compilation} \\ \Lambda' = \left(\mathcal{L}_{S}, \mathcal{L}_{Q} = \mathsf{patt}_{1}\right) \end{array}$$

#### Suppressing Query Equality



$$\begin{array}{c} \text{STE} \\ \Lambda = \begin{pmatrix} \mathcal{L}_{\text{S}}, \mathcal{L}_{\text{Q}} = & \\ \end{pmatrix}, \text{patt} \end{pmatrix} \longrightarrow \begin{array}{c} \text{Cache-Based} \\ \text{Compiler} \\ \text{(CBC)} \end{array} \longrightarrow \begin{array}{c} \Lambda' = \begin{pmatrix} \mathcal{L}_{\text{S}}, \mathcal{L}_{\text{Q}} = \text{nrp} \end{pmatrix} \end{array}$$

$$\mathsf{patt} = \left\{ \begin{array}{c} \mathsf{nrp} \\ \bullet \\ \bullet \end{array} \right.$$

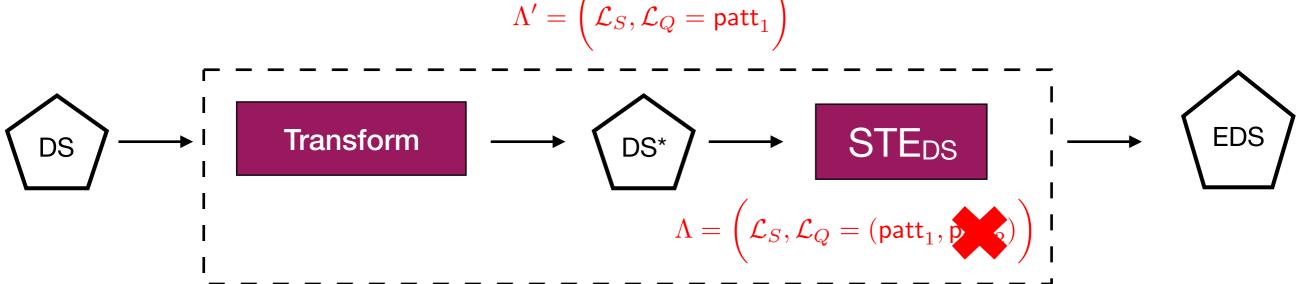
#### Leakage Suppression

#### Through Transformation





$$\Lambda' = igg(\mathcal{L}_S, \mathcal{L}_Q = \mathsf{patt}_1igg)$$



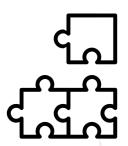


$$\begin{array}{c} \textbf{STE} \\ \mathcal{L}_{Q} = (\mathsf{qeq}, \mathsf{patt}) \\ \\ \mathsf{patt} = \left\{ \begin{array}{c} \mathsf{nrp} \\ \mathsf{rp} \end{array} \right. \end{array} \qquad \begin{array}{c} \textbf{STE'} \\ \mathcal{L}_{Q} = \mathsf{nrp} \end{array}$$

- Cache-based Compiler (CBC)
  - suppresses the query equality and the repeating sub-pattern
  - induces an additive poly-log overhead
  - Requires a rebuildable STE



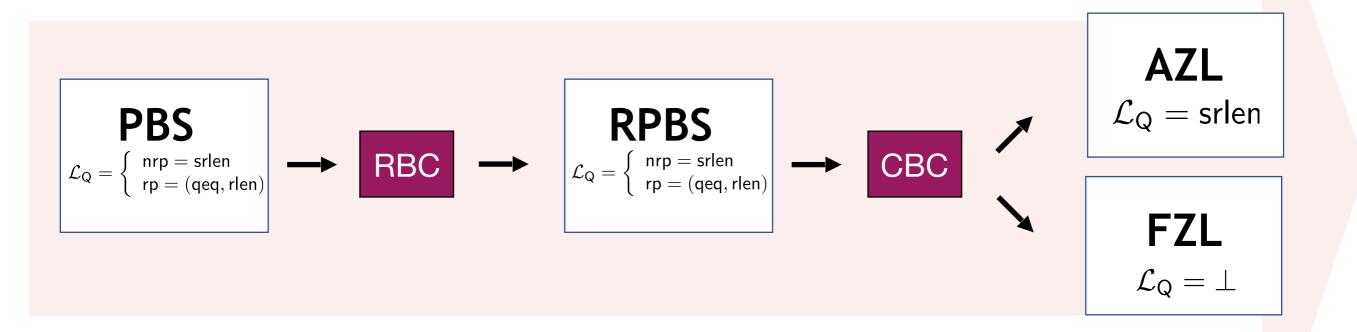
- Rebuild Compiler (RBC)
  - makes any STE scheme rebuildable
  - preserves the scheme's query efficiency
  - adds a super-linear rebuild cost



$$\begin{array}{c} \textbf{STE} \\ \mathcal{L}_{Q} = (\mathsf{qeq}, \mathsf{patt}) \\ \mathsf{patt} = \left\{ \begin{array}{c} \mathsf{nrp} \\ \mathsf{rp} \end{array} \right. \end{array} \longrightarrow \begin{array}{c} \textbf{RSTE} \\ \mathcal{L}_{Q} = (\mathsf{qeq}, \mathsf{patt}) \\ \mathsf{patt} = \left\{ \begin{array}{c} \mathsf{nrp} \\ \mathsf{rp} \end{array} \right. \end{array} \longrightarrow \begin{array}{c} \textbf{CBC} \end{array} \longrightarrow \begin{array}{c} \textbf{RSTE'} \\ \mathcal{L}_{Q} = \mathsf{nrp} \end{array}$$

The problem boils down to reduce nrp of the base STE scheme

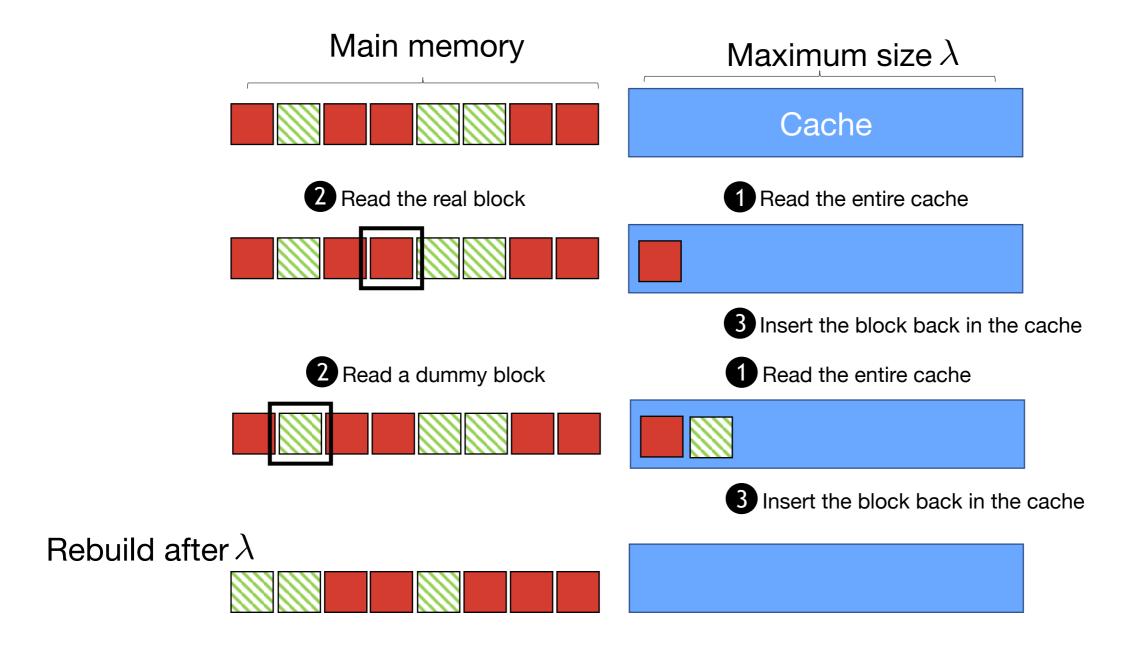


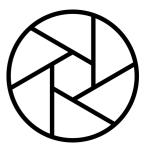


- Piggyback scheme (PBS)
  - hides the response length for non-repeating queries
  - introduces query latency

#### Square-Root ORAM [GO96]



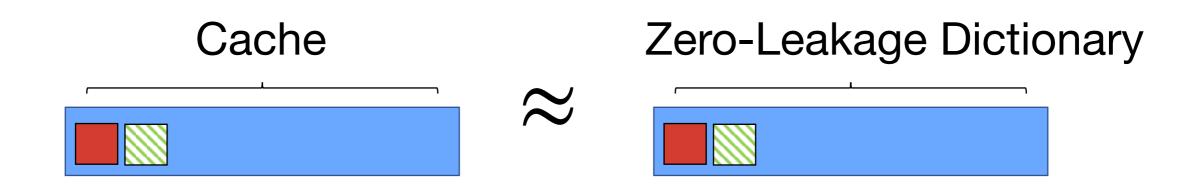




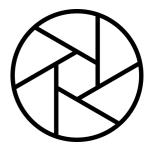


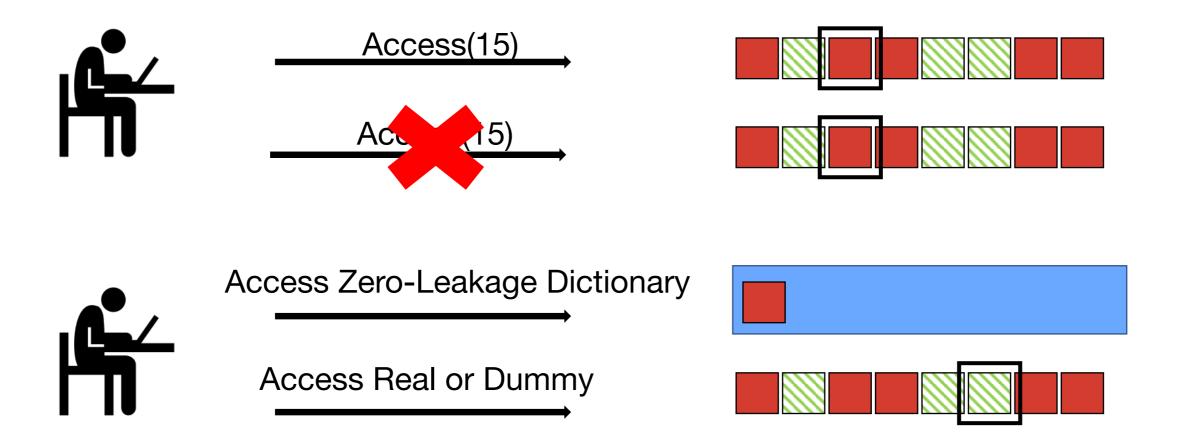
- Main memory is an encrypted array construction
- Accessing element is done deterministically through PRP evaluation
- Adversary learns if/when an access to the same element is repeated
  - Leaks query equality





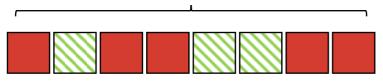
- The cache is an encrypted dictionary data structure
  - Given a label, it outputs an element or ⊥
- The cache is accessed in its entirety
  - Most trivial zero-leakage dictionary construction; therefore no query leakage











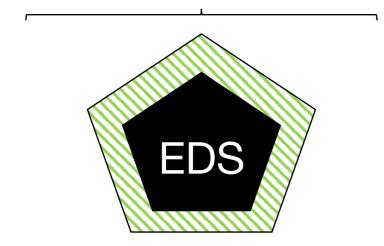
Zero-Leakage Dictionary



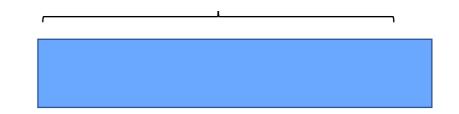
K 7

K 7

#### **Encrypted Data Structure**



#### Zero-Leakage Dictionary



#### Requirements

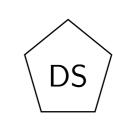


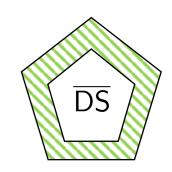


- Data structure has to be extendable and safe
- Base scheme has to have smaller non-repeating sub-pattern



#### Data Structure Extension



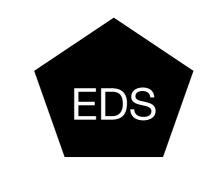


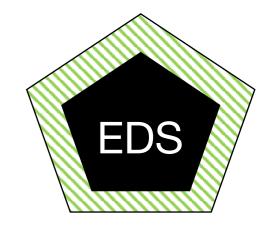
- $\lambda$ -extension:
  - Extend the query space of the data structure with  $\lambda$  dummies
  - $\forall q \in \overline{\mathbb{Q}} \setminus \mathbb{Q} \text{ s.t. } \mathbb{Q} \subseteq \overline{\mathbb{Q}}$

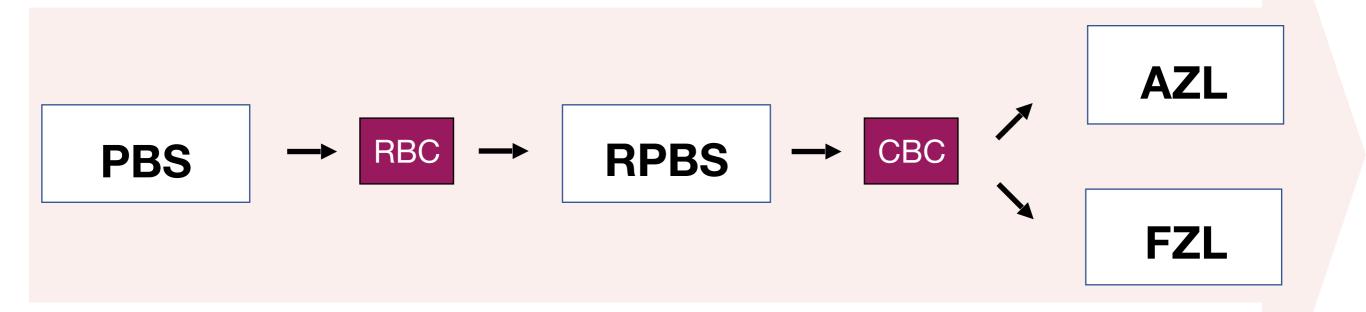
$$\mathsf{Query}\bigg(\overline{\mathsf{DS}},q\bigg) = \bot$$

• Safe  $\lambda$ -extension:

$$\mathcal{L}_{\mathsf{S}}(\overline{\mathsf{DS}}) \leq \mathcal{L}_{\mathsf{S}}(\mathsf{DS})$$
 $\mathcal{L}_{\mathsf{Q}}(\overline{\mathsf{DS}},q) \leq \mathcal{L}_{\mathsf{Q}}(\mathsf{DS},q)$ 

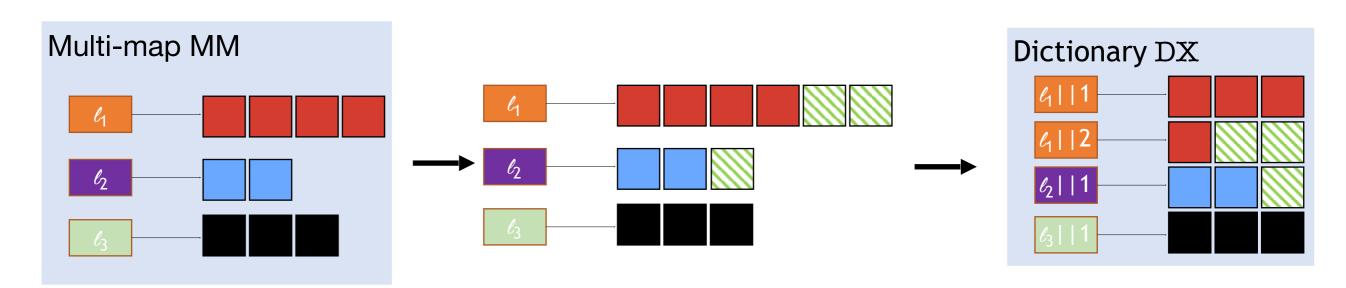




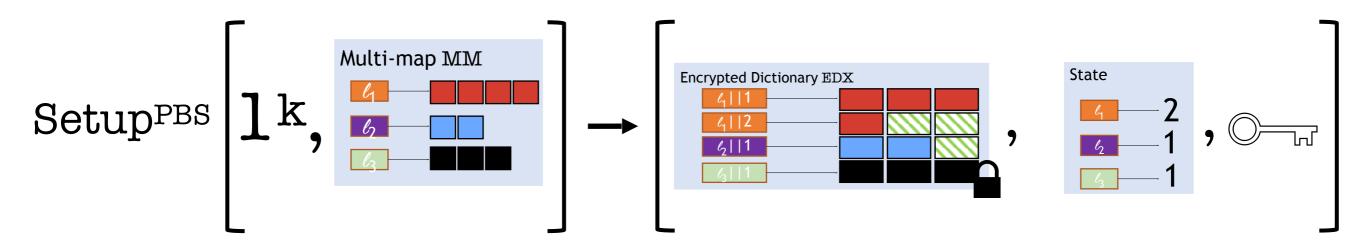


#### PBS: Data transformation

- Batch size (ex:  $\alpha$  = 3)
- Pad all responses to a multiple of  $\alpha$



#### **PBS Details**

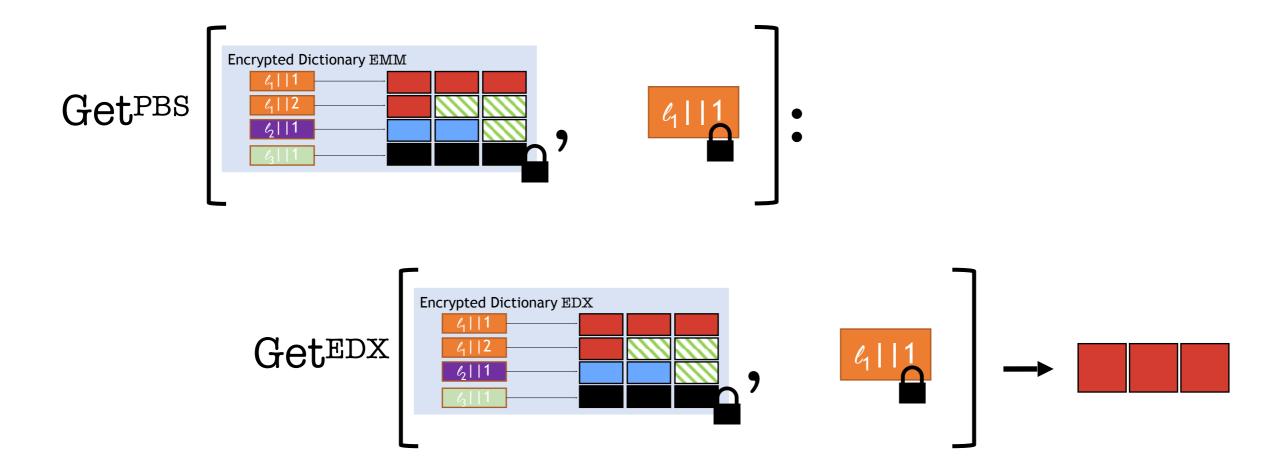


#### **PBS** Details

• Consider a sequence of labels  ${f q}=(\ell_1,\ell_2)$ 

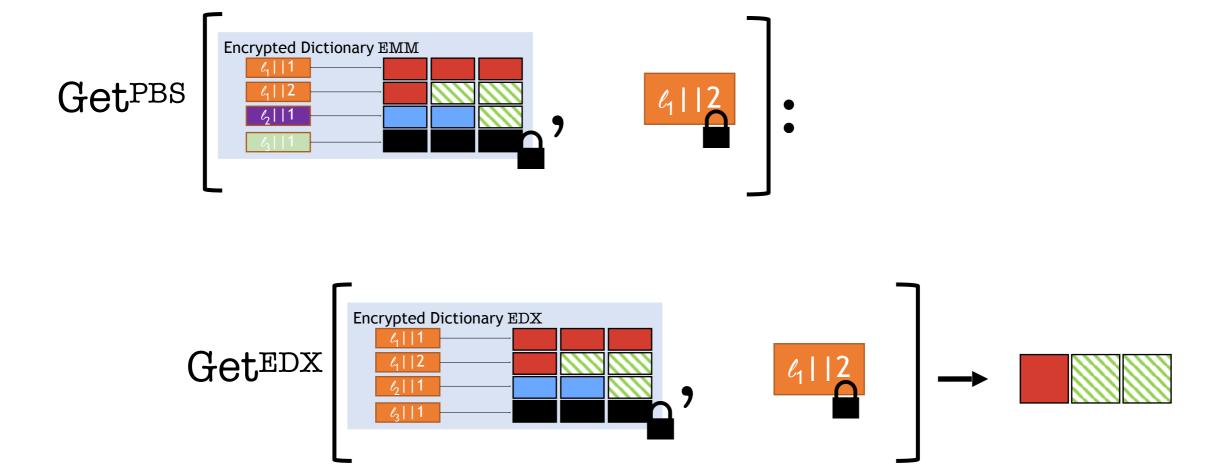
- 1. las 2 batches
- 2. Instantiate a queue 4111 4112
- 3. Compute

4. Update queue 4112



- 1. 6 has 1 batch
- 2. Update the queue 4112 4111
- 3. Compute

4. Update queue 6111

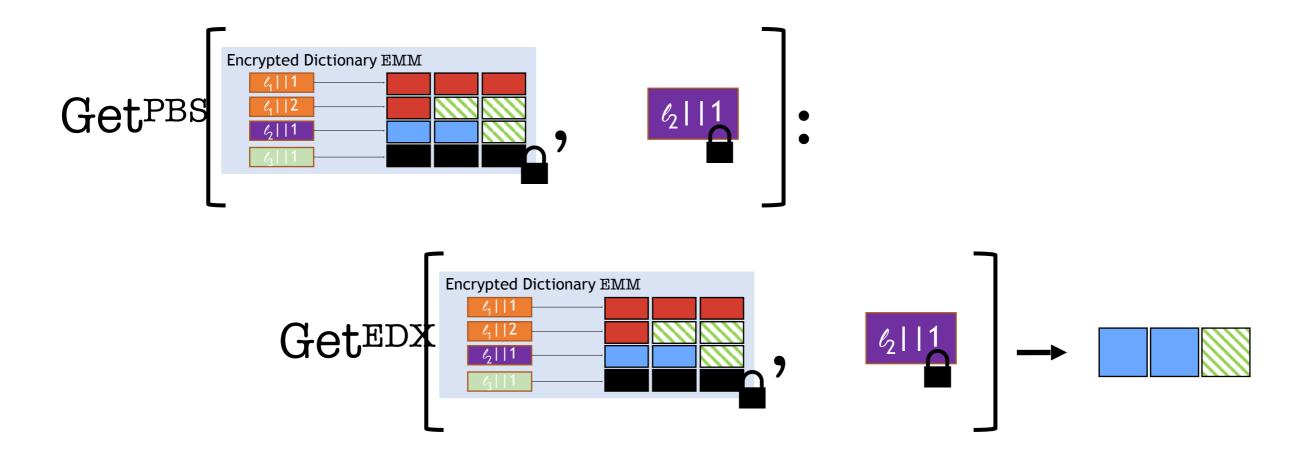


Token PBS 
$$\begin{bmatrix} \bigcirc & \\ \bigcirc & \\ \bigcirc & \\ \bigcirc & \\ & \bigcirc & \end{bmatrix}$$
,  $\begin{bmatrix} \bot & \\ & & \\ & & \\ & & \end{bmatrix}$ .

1. Compute

Token 
$$\left[ \bigcirc, \boxed{2} \right]$$

2. Update queue



## **PBS Latency**

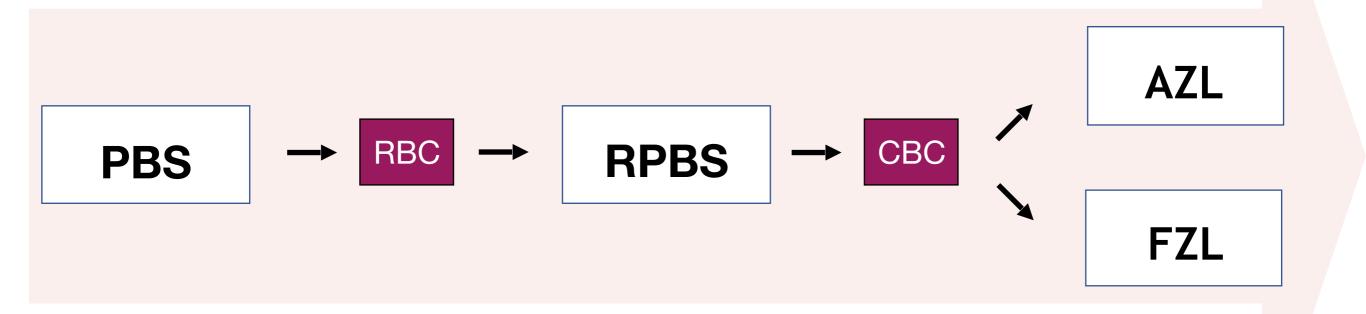
The worst-case query sequence of size t has latency

$$t \cdot \left(\frac{\max_{r \in \mathbb{R}_{DS}} |r|_w}{\alpha} - 1\right)$$

Real-world sequences have latency

$$\varepsilon \cdot t$$
 with probability at least 
$$1 - \exp\left(-2t \Big(\varepsilon \cdot \frac{\alpha}{\max_{r \in \mathbb{R}_{\mathrm{DS}}} |r|_w}\Big)^2\right)$$

where queries are drawn from a Zipf distribution and longer responses are mapped to less frequent labels



## **AZL** Analysis

• Worst-case query complexity over  $\lambda$  queries

$$\sum_{i=1}^{\lambda} \mathsf{T}^{\mathsf{eds}}(q_i) + O\left(\lambda \cdot \max_{q \in \mathbf{q}} |r|_w \cdot \log^2 \lambda\right) + O\left(\sum_{r \in \mathbb{R}_{\mathsf{DS}}} |r|_w \cdot \log^2 \# \mathbb{Q}_{\mathsf{DS}}\right)$$

• Comparison to ORAM simulation (Path-ORAM [SvDSFRD13])

$$\mathsf{T}^{\mathsf{eds}}(q_1,\ldots,q_{\lambda}) = o\left(\mathsf{T}^{\mathsf{tree}}(q_1,\ldots,q_{\lambda})\right)$$

when

Natural Assumption:

If response lengths are power –law distributed

$$\sum_{r \in \mathbb{R}_{\mathsf{DS}}} |r|_w = o\left(\sum_{i=1}^{\lambda} B(q_i) \cdot \max_{r \in \mathbb{R}_{\mathsf{DS}}} |r|_w\right) \text{ and } \lambda \cdot \max_{q \in \mathbf{q}} |\mathsf{qu}(\mathsf{DS},q)|_w = o\left(\sum_{r \in \mathbb{R}_{\mathsf{DS}}} |r|_w\right)$$

# Part 2\*

## Suppressing Volume

\*joint work with Seny Kamara

https://eprint.iacr.org/2018/978

## Leakage Suppression

## Through Compilation



$$\begin{array}{c} STE \\ \Lambda = \left(\mathcal{L}_{S}, \mathcal{L}_{Q} = \left(\mathsf{patt}_{1}, \mathsf{p}\right)\right) \end{array} \longrightarrow \begin{array}{c} \mathsf{Compilation} \\ \Lambda' = \left(\mathcal{L}_{S}, \mathcal{L}_{Q} = \mathsf{patt}_{1}\right) \end{array}$$

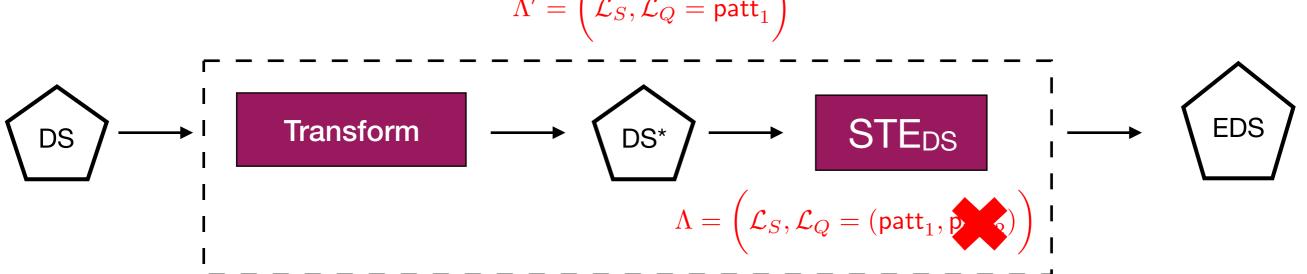
## Leakage Suppression

## Through Transformation





$$\Lambda' = igg(\mathcal{L}_S, \mathcal{L}_Q = \mathsf{patt}_1igg)$$

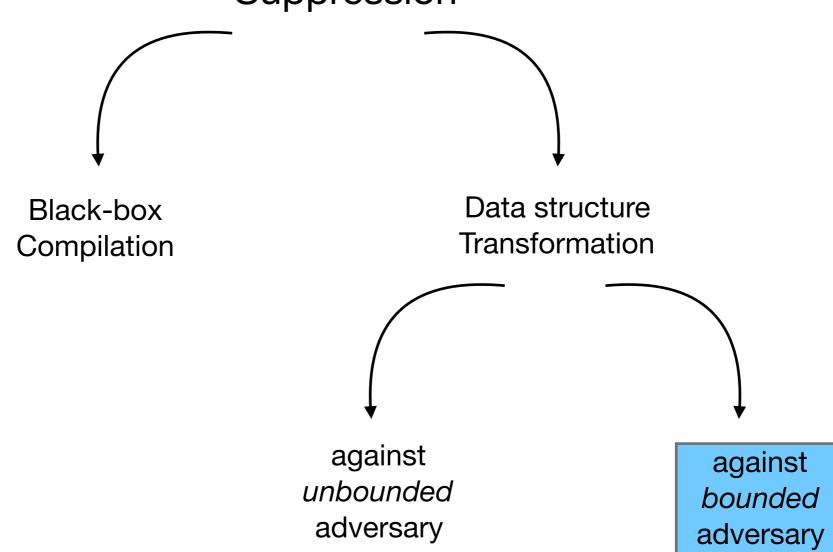




: is there any other approach to suppress leakage?



#### Suppression



## Computationally-Secure Leakage



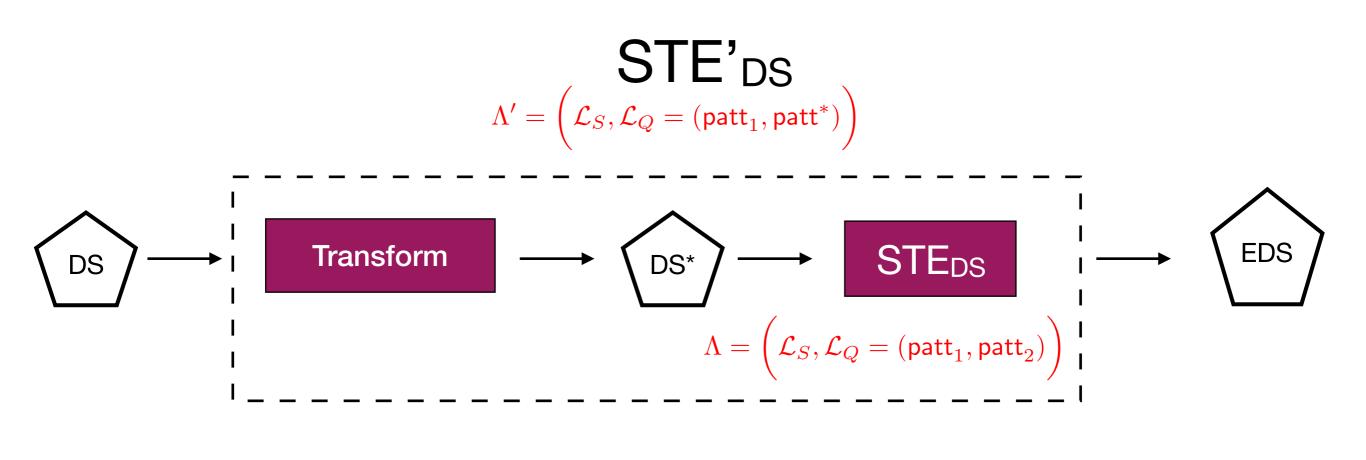




Unbounded Adversary vs. Bounded Adversary

## Leakage Suppression [KMO18]

Through Transformation



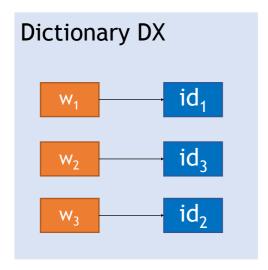


: can we suppress the response length pattern?

## Background

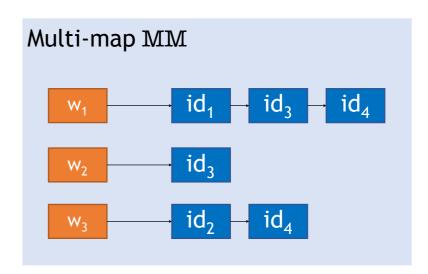
Dictionary and Multi-Map data structures

DXs map labels to values



• Get: DX[w<sub>3</sub>] returns id<sub>2</sub>

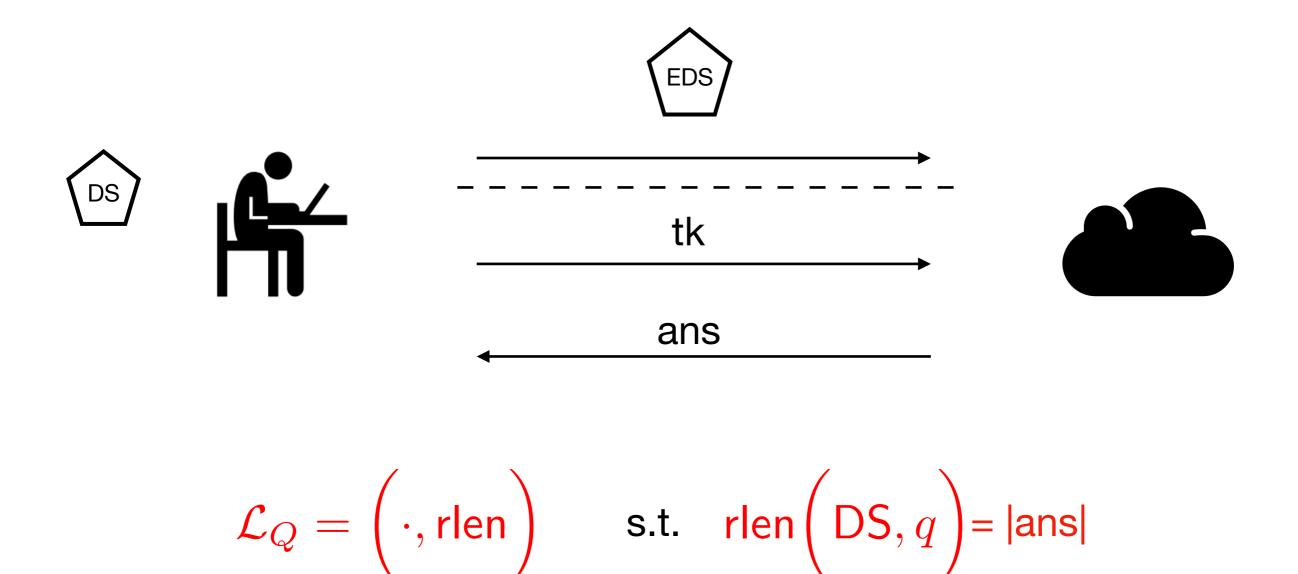
MMs map labels to tuples



Get: MM[w<sub>3</sub>] returns (id<sub>2</sub>, id<sub>4</sub>)

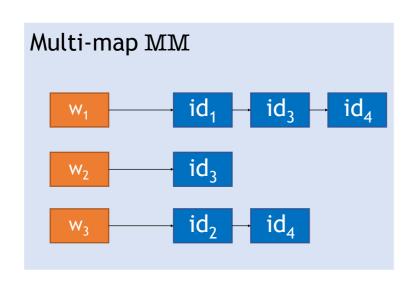
## Background

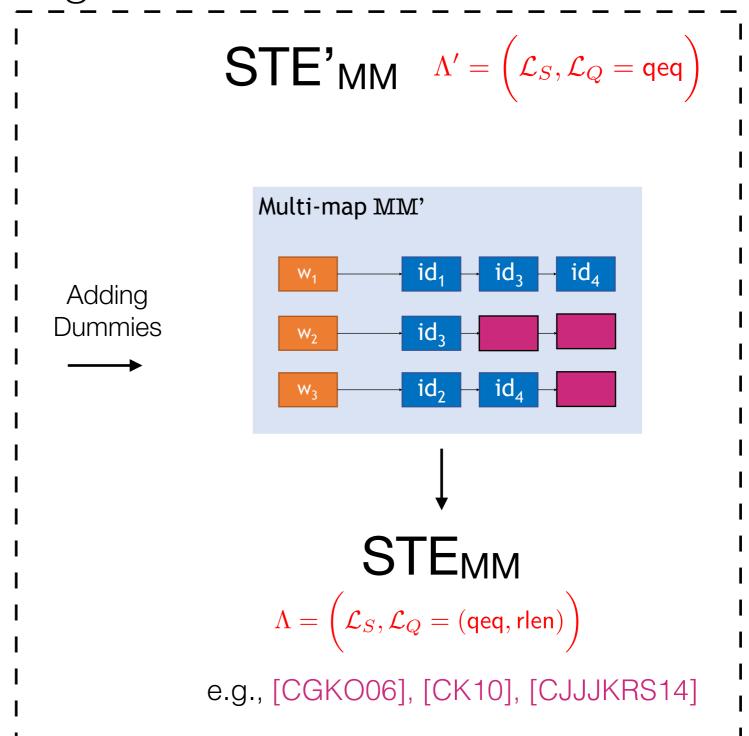
Response Length Pattern (rlen) or Volume Pattern



## Naive Approaches to Hide Volume

Through Naive Padding





## Naive Approach to Hide Volume

## Through Naive Padding

Query complexity

$$O(\max_{\ell \in \mathbb{L}_{\mathsf{MM}}} \# \mathsf{MM}[\ell])$$

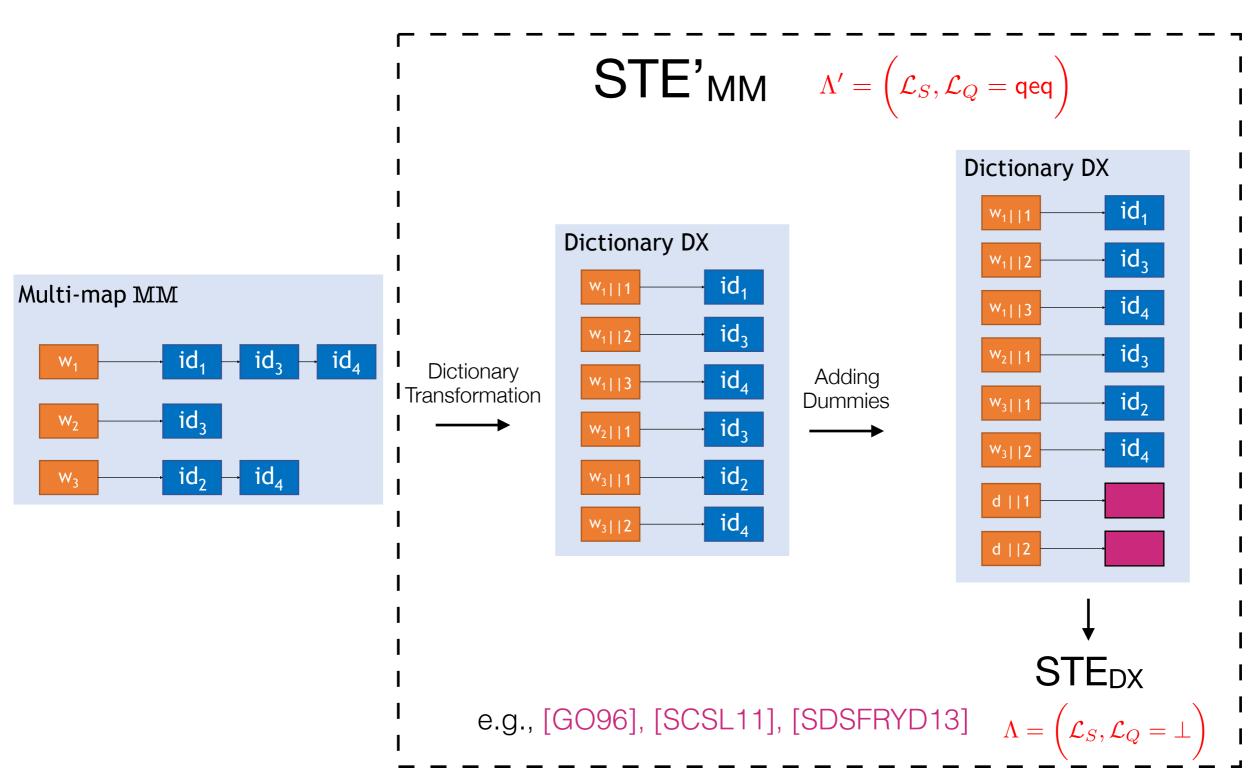
Storage complexity

$$O(\#\mathbb{L}_{\mathsf{MM}} \cdot \max_{\ell \in \mathbb{L}_{\mathsf{MM}}} \#\mathsf{MM}[\ell])$$

Non-interactive

## Naive Approach to Hide Volume

Through Leakage-Free Dictionary



## Naive Approach to Hide Volume

Through Leakage-Free Dictionary (w/ [SDSFRYD13])

Query complexity

$$O\bigg(\max_{\ell \in \mathbb{L}_{\mathsf{MM}}} \#\mathsf{MM}[\ell] \cdot \log^2 \bigg(\sum_{\ell \in \mathbb{L}_{\mathsf{MM}}} \#\mathsf{MM}[\ell]\bigg)\bigg)$$

Storage complexity

$$O\left(\sum_{\ell \in \mathbb{T}_{\mathsf{AMM}}} \#\mathsf{MM}[\ell]\right)$$

Interactive



: can we achieve the best of both worlds?

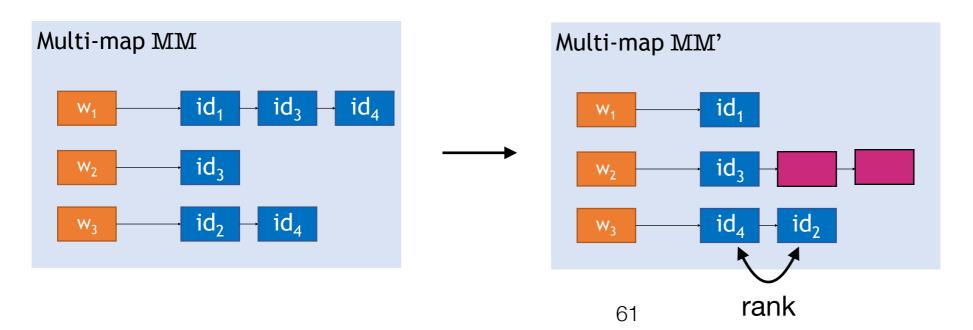
#### Contributions

- Pseudo-Random Transform (PRT)
- Volume Hiding Multi-Map Encryption scheme (VLH)
- Densest-Subgraph Transform (DST)
- Advanced Volume Hiding Multi-Map Encryption scheme (AVLH)
- Dynamism

## Pseudo-Random Transform (PRT)

- Pseudo-random function  $F: \{0,1\}^k \times \{0,1\}^* \to \{0,1\}^{\log \nu}$
- Minimum response length  $\lambda$
- Replace the response length of  $\ell$  by  $\lambda + F_K(\ell | \# MM[\ell])$ 
  - Truncate if  $\lambda + F_K(\ell || \#\mathsf{MM}[\ell]) \leq \#\mathsf{MM}[\ell]$
  - Pad if  $\lambda + F_K(\ell || \#\mathsf{MM}[\ell]) > \#\mathsf{MM}[\ell]$
- Rank the response identities

E.g., 
$$\lambda = 1$$
 and  $\nu = 3$ 



$$F_K(w_1||3) = 0$$
  
 $F_K(w_2||1) = 2$   
 $F_K(w_3||2) = 1$ 



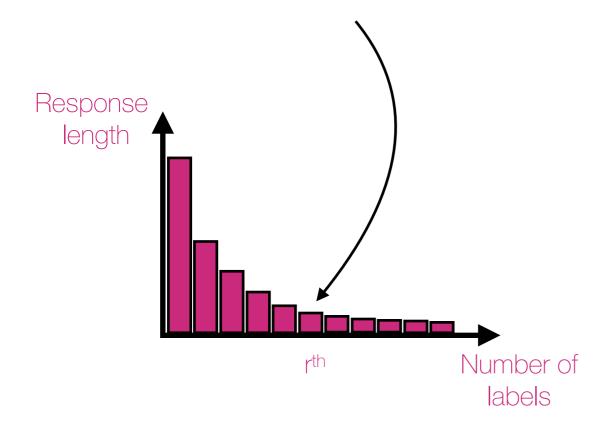
: what about the number of truncations and storage overhead?

## Pseudo-Random Transform (PRT)

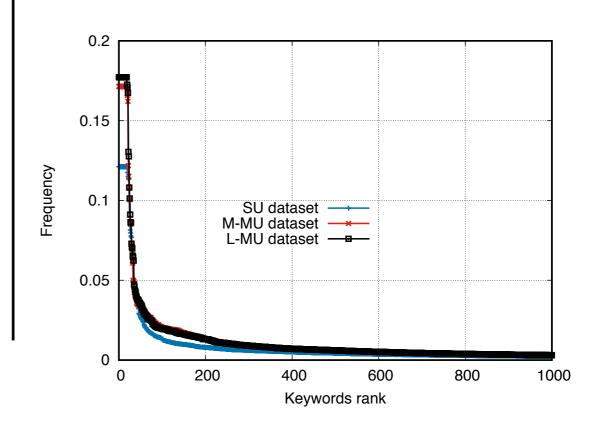
## Zipf-Distributed MM

A MM is Zipf-distributed if the rth response has length:

$$\frac{1}{r \cdot H_{\#\mathbb{L}_{\mathsf{MM}},1}} \cdot \sum_{\ell \in \mathbb{L}_{\mathsf{MM}}} \#\mathsf{MM}[\ell]$$



- Common in real-world datasets
   [Zipf35], [CCKS07]
- Ex: Enron 0.5M emails (2004)



## Pseudo-Random Transform (PRT)

## Analysis

- Let  $\alpha$  be the storage reduction multiplicative factor
- If for  $1/2 < \alpha < 1$ , then with probability at least
  - $1 \exp\left(-\#\mathbb{L}_{\mathsf{MM}} \cdot (2\alpha 1)^2/8\right)$  the size of the MM is at most

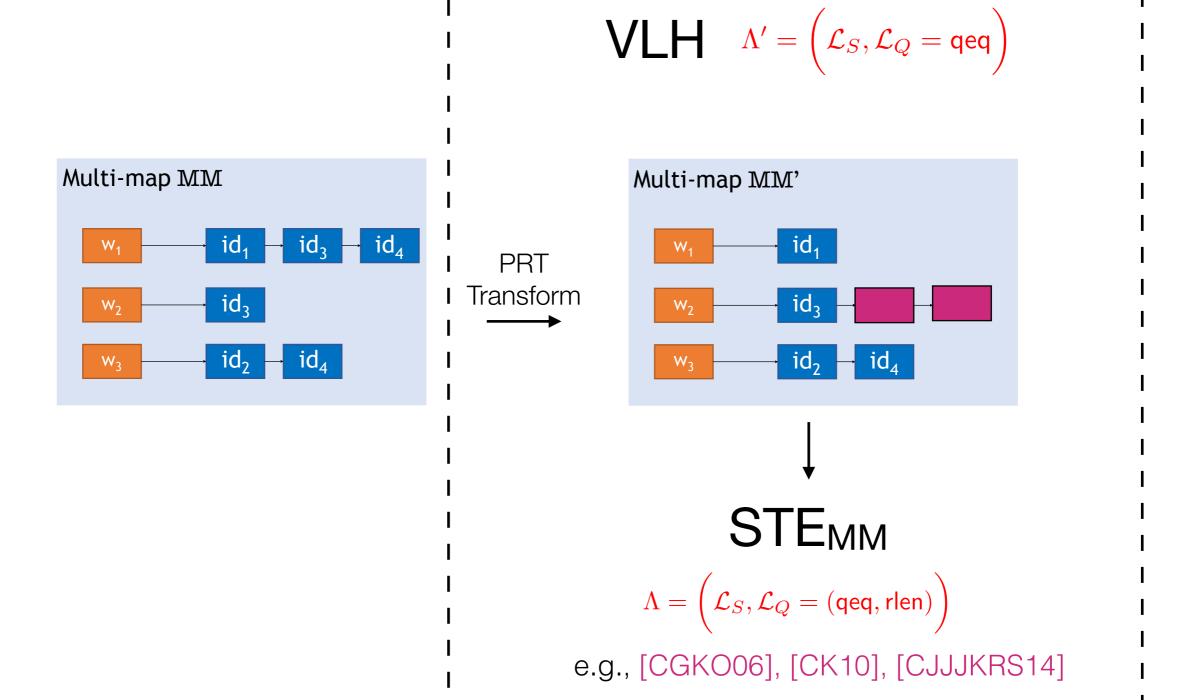
$$\alpha \cdot \# \mathbb{L}_{\mathsf{MM}} \cdot \max_{\ell \in \mathbb{L}_{\mathsf{MM}}} \# \mathsf{MM}[\ell]$$

•  $1 - \exp\left(-2\#\mathbb{L}_{\mathsf{MM}} \cdot \log^2(\#\mathbb{L}_{\mathsf{MM}})\right)$  the number of truncations is at most

$$\frac{1}{\log(\#\mathbb{L}_{\mathsf{MM}})} \cdot \#\mathbb{L}_{\mathsf{MM}}$$

## Volume Hiding EMM (VLH)

## Design



## Volume Hiding EMM (VLH)

Analysis (with standard EMMs)

Query complexity (worst-case)

$$O(\lambda + \nu)$$

Storage complexity

$$O(\lambda \cdot \# \mathbb{L}_{\mathsf{MM}} + \sum_{\ell \in \mathbb{L}_{\mathsf{MM}}} n_{\ell})$$
 s.t.  $n_{\ell} \xleftarrow{\$} [\nu]$ 

and w.h.p. when  $1/2 < \alpha < 1$ 

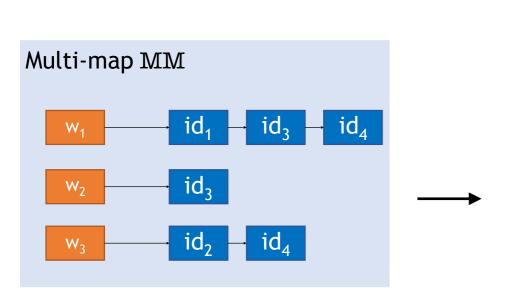
$$O(\alpha \cdot (\nu - 1) \cdot \# \mathbb{L}_{\mathsf{MM}})$$

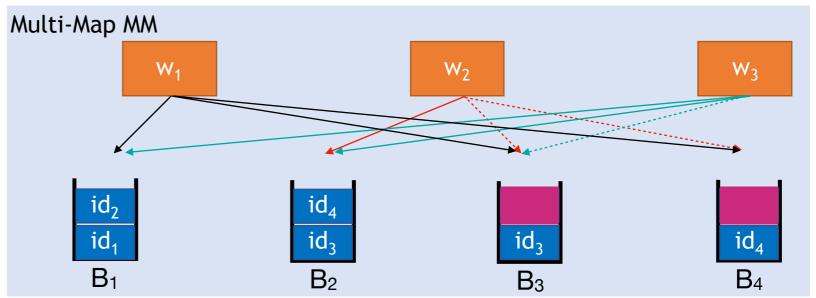
- Non-Interactive
- Lossy

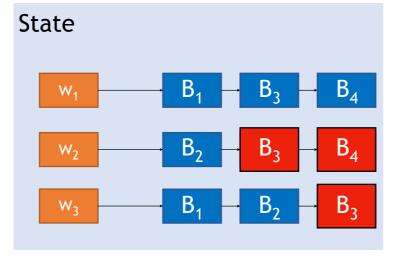
#### Overview

- We view a MM as a bi-partite graph  $G = (\mathbb{L}_{MM}, \mathbf{B}), E$ 
  - top vertices: labels L<sub>MM</sub>
  - bottom vertices: bins B
- Given MM we build a Erdös-Rényi random graph
- All labels in MM have the same number of edges
- Goal: given a label, fetch the same number of bins
  - reduce the load of the bin

#### Details







Storage overhead

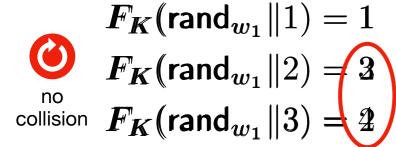
 $O(\#\mathbb{L}_{\mathsf{MM}} \cdot \max_{\ell \in \mathbb{L}_{\mathsf{MM}}} \#\mathsf{MM}[\ell])$ 

Similar to Naive Padding

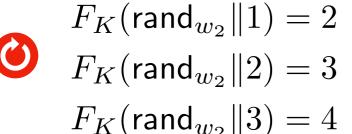
#### Details

#### **Edge Generation**

$$\mathsf{rand}_{w_1} \xleftarrow{\$} \{0,1\}^k$$



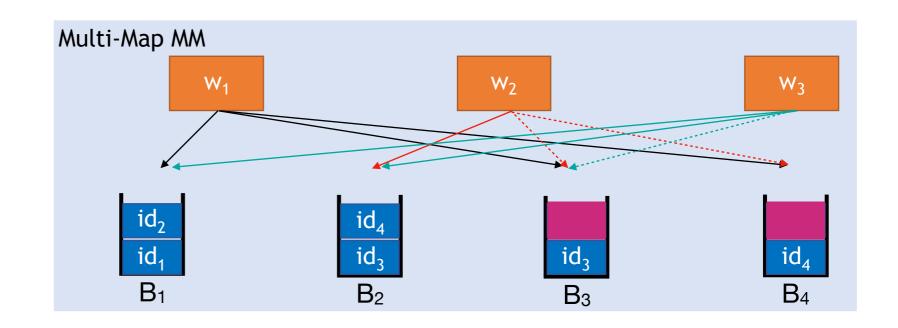
$$\mathsf{rand}_{w_2} \xleftarrow{\$} \{0,1\}^k$$

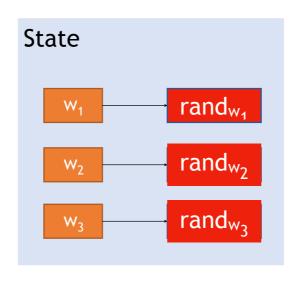


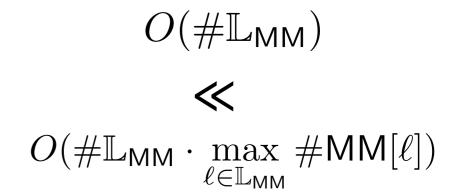
$$\mathsf{rand}_{w_3} \xleftarrow{\$} \{0,1\}^k$$

$$F_K(\text{rand}_{w_3}||1) = 1$$

 $F_K(\operatorname{rand}_{w_3} || 2) = 2$   $F_K(\operatorname{rand}_{w_3} || 3) = 3$ 

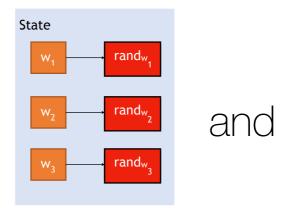


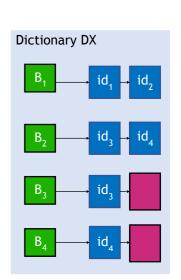




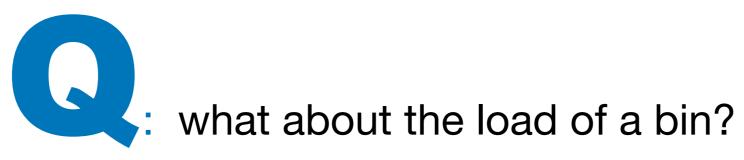
#### Details

• The output of DST is equal to: O,





- To fetch a keyword w, retrieve randw, from the state
- Compute bins' identifiers  $F_K(\operatorname{rand}_{w_1}||1)$ ,  $F_K(\operatorname{rand}_{w_1}||2)$ ,  $F_K(\operatorname{rand}_{w_1}||3)$
- Retrieve all the bins from the dictionary DX



## Analysis

With probability at least  $1-\varepsilon$  , the load of a bin is

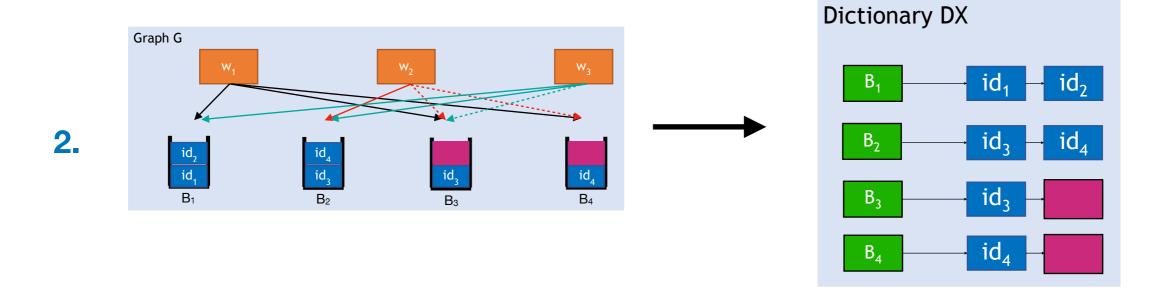
$$\frac{N}{n} + \frac{\ln(1/\varepsilon)}{3} \left( 1 + \sqrt{1 + \frac{18N}{n \cdot \ln(1/\varepsilon)}} \right)$$

where 
$$N = \sum_{\ell \in \mathbb{L}_{\mathsf{MM}}} \# \mathsf{MM}[\ell]$$

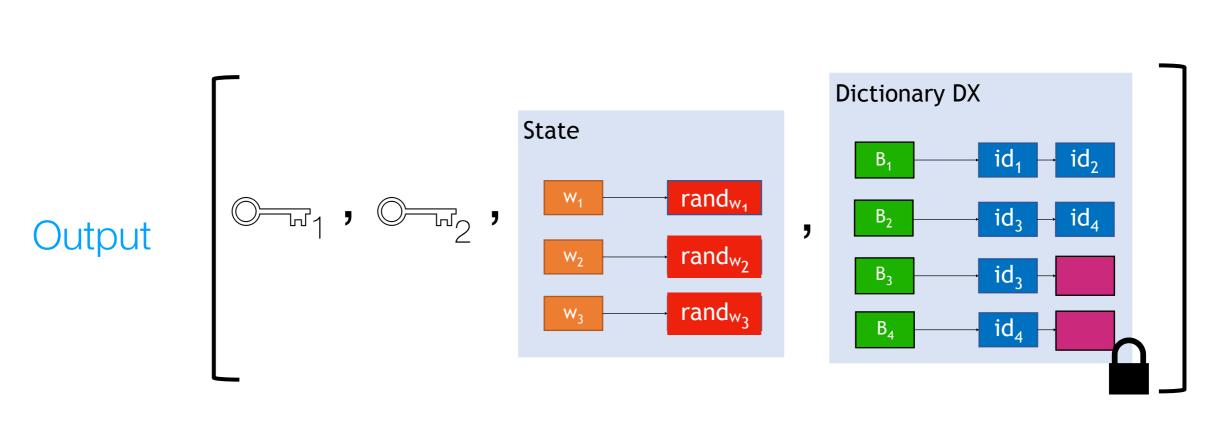
The size of the transformed multi-map MM is O(N)

The size of the state is  $O(\# \mathbb{L}_{MM}) \iff O(N)$ 

Setup (1)



Setup (2)



Token

- 1. Fetch randw, from State
- 2. Compute  $\mathbf{t} = \left(F_K(\mathsf{rand} \| i)\right)_{i \in [3]}$
- 3. for each identifier i in tadd to tk

EDX.Token 
$$\left[ \bigcirc , i \right] \longrightarrow tk_t$$

Output tk

Query

1. for each sub-token tki in tk

EDX.Query 
$$\begin{bmatrix} tk_i, & Dictionary DX \end{bmatrix} \longrightarrow ct_i$$

Output 
$$ct = (ct_1, ct_2, ct_3)$$

Analysis ([CGKO06])

Query complexity w.h.p.

$$O\left(t \cdot \frac{N}{\#\mathbb{L}_{\mathsf{MM}} \cdot \mathsf{polylog}(\#\mathbb{L}_{\mathsf{MM}})}\right)$$

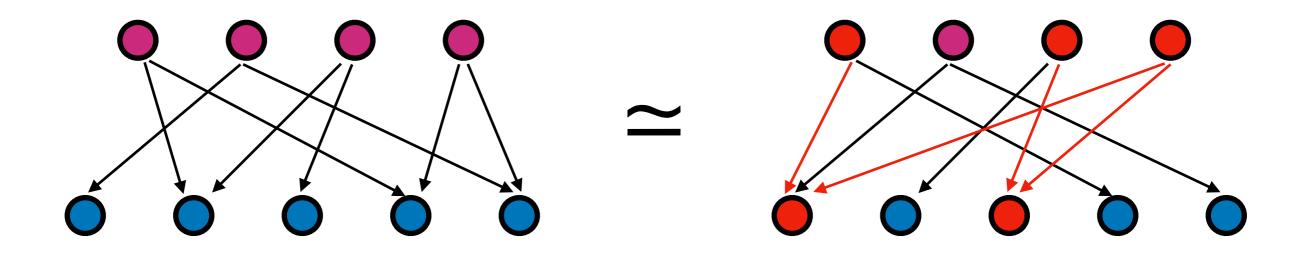
where t is the maximum length and  $N = \sum_{\ell \in \mathbb{L}_{\text{MM}}} \#\text{MM}[\ell]$ 

Storage complexity w.h.p.

- Non-Interactive
- Non-Lossy

#### Densest-Subgraph Transform (DST)

Improving Storage



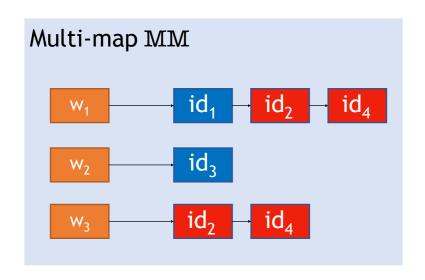
Erdös-Rényi graph

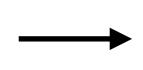
Erdös-Rényi graph with planted dense subgraph

Found applications in public-key cryptography [ABW10] and computational complexity of financial products [ABBG11]

#### Densest-Subgraph Transform (DST)

#### Improving Storage





 $w_1$   $w_2$   $w_3$   $w_4$   $id_2$   $id_4$   $id_3$ 

Concentrated MM: labels with non-empty intersection

id<sub>2</sub> and id<sub>4</sub> constitute the concentrated part

Add the concentrated part only once to the graph

Result: Reduce the load of bins

#### Densest-Subgraph Transform (DST)

#### Analysis

With probability at least  $1-\varepsilon$  , the load of a bin is

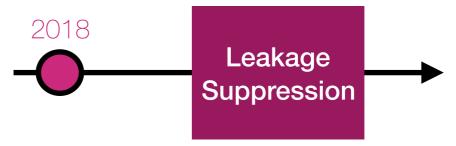
$$\frac{N - N_{\text{DS}}}{n} + \frac{\ln(1/\varepsilon)}{3} \left( 1 + \sqrt{1 + \frac{18(N - N_{\text{DS}})}{n \cdot \ln(1/\varepsilon)}} \right)$$

where  $N_{\mathrm{DS}}$  is the size of the concentrated part.

#### Instead of

$$\frac{N}{n} + \frac{\ln(1/\varepsilon)}{3} \left( 1 + \sqrt{1 + \frac{18N}{n \cdot \ln(1/\varepsilon)}} \right)$$

• Introduce a new direction in encrypted search



- A general framework that suppresses the search pattern
- First solution to hide response-length pattern (volume pattern)
- A general compiler that makes any STE scheme rebuildable
- First scheme to leak at most the sequence response length (very hard to exploit)
- The first scheme that leaks (nothing)
- Introduces a new tradeoff: query latency vs. security

- Volume pattern has been recently leveraged as an attack vector [KKNO16], [GLMP18]
- · Without trivial naive padding, hiding volume is extremely hard
- Hiding volume is an important step for leakage suppression



- The first non-trivial schemes that hide the volume pattern
  - VLH based on a new lossy pseudo-random transform (PRT)
  - AVLH based on a new non-lossy densest-subgraph transform (DST)

- Leveraging computational assumptions to suppress leakage
  - Intuitively it is hard to hide volume information theoretically without padding
  - Get around this leveraging computational assumptions
  - first to do so for any pattern, and for volume in particular
  - possibility to leverage computational assumptions to suppress other patterns
- Introducing a new tradeoff: correctness vs. security
- Hiding volume can help thwart many existing attacks: [IKK12], [CGPR15], [KKNO16], [LMP18], [GLMP18], [LMP19]

## Thank you!